

Robust Technique for Quantification of NMR Imaging Data

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Nuclear magnetic resonance spectroscopy and imaging methods provide unprecedented opportunities for characterizing the transport of fluids in porous media (Chen et al., 1993, 1994). A key challenge in the application of NMR methods is the analysis of measured signals to determine properties used for describing the storage and flow of fluids. One of the basic measures is the amount of observed fluid as a function of location. Such information can be used to determine the porosity distribution when the medium is saturated with a single fluid phase, or fluid saturations when multiple fluid phases are present.

In NMR experiments, the intrinsic magnetization intensity is proportional to fluid content and can be used to determine the porosity and saturation (Mandava et al., 1990). However, fluids in porous media tend to have very short relaxation times, so significant relaxation occurs before the magnetization intensity signal is acquired. If the relaxation process is suitably modeled, accurate estimates of the intrinsic magnetization intensity, and hence local fluid content, can be made (Chen et al., 1993, 1994). Previously, multiexponential (Chen et al., 1993, 1994) or stretched exponential (Fordham et al., 1993) models have been used to model the relaxation process. These have required the solution of associated nonlinear regression problems to obtain the values of several discrete parameters within the models. Algorithms to solve these problems may be very computer intensive, and may fail to converge to the global minimum or at all. We introduce the use of a continuous distribution for representing the relaxation rates. This can provide a more accurate representation of the relaxation process (Brownstein and Tarr, 1977; Liaw et al., 1995). Furthermore, the analysis is robust and the model is linear in the parameters so that the global optimum is computed directly.

While the same principles may be used for quantification of measurements which are resolved with zero, one, two, or three spatial variables, we will confine our attention to one-dimensional imaging. This is consistent with previously reported quantitative applications, which have largely been directed to resolutions with a single spatial variable. We will demonstrate the methodology with images obtained using a Hahn spin echo (Chen et al., 1993). Other imaging sequences, such as CPMG, can be used as well.

Background

In the NMR profile imaging experiments, the spatial resolution is obtained by applying a magnetic field gradient G in one direction. In this technique, the spatial information is represented in the frequency domain since the Larmor frequency ω is proportional to the strength of the magnetic field

$$\omega = \gamma Gz, \quad (1)$$

where γ is the proton gyromagnetic ratio. The use of the NMR profile imaging techniques to detect fluid saturation is based on the fact that the intrinsic magnetization $M_0(\omega)$ is proportional to the number of proton spins. Within a pixel located along the z direction and centered at z , the intrinsic magnetization is given by

$$M_0(\omega) = k(z)N(z)\Delta z/\Delta \omega, \quad (2)$$

where

$$N(z) = \int \int_{A_{xy}} \rho(x, y, z) dx dy \quad (3)$$

is the proton linear density in the sample. A_{xy} is the cross-sectional area in the xy plane, and ρ is the proton density in the sample. The calibration constant $k(z)$ is determined by use of a reference sample (Chen et al., 1993; Chang et al., 1993). The fluid saturation distribution can then be calculated by

$$S(z) = N(z)/N_i(z), \quad (4)$$

where $N_i(z)$ is the linear proton density measured when the sample is fully saturated (such as prior to displacement). The porosity distribution $\phi(z)$ can also be determined by

$$\phi(z) = N_i(z)/\rho A_{xy}, \quad (5)$$

where ρ is the proton density of the observed fluid phase.

The main difficulty in quantification arises from the fact that the intrinsic magnetization intensity is not measured since the equilibrium magnetization, which is oriented along the direction of the static field, must be shifted into the orthogonal plane to be observed. For fluids in porous media, significant relaxation may occur before the magnetization intensity can be observed. The intrinsic magnetization intensity can be estimated from a model of the relaxation process. Parameters within the relaxation model can be estimated from data measured at different times, for which different degrees of relaxation have occurred. The estimation of parameters is best done on a local (pixel-by-pixel) basis since relaxation depends on properties which may vary spatially.

Previously, a multiexponential model (Chen et al., 1993, 1994) and stretched exponential model (Fordham et al., 1993) have been used for representing the magnetization intensity evolution. Profile images were acquired with a Hahn spin echo using several different values for echo time TE . With the multiexponential representation, the observed intensity of magnetization corresponding to the i th pixel is represented by

$$M(\omega_i, TE) = \sum_{j=1}^{N_c} M_{0j}(\omega_i) \exp(-TE/T_{2ij}). \quad (6)$$

With the stretched exponential model, the magnetization intensity is represented as

$$M(\omega_i, TE) = M_0(\omega_i) \exp[-(TE/T_{2ia})^\alpha] \quad (7)$$

where α is the stretch exponent, and T_a is the spin-spin relaxation time. These representations allow for the use of different relaxation parameters at different pixels. Estimates of the intensities and the relaxation parameters are obtained by nonlinear regression through a minimization of an objective function

$$J_i = \sum_{l=1}^n [M^{\text{obs}}(\omega_i, TE_l) - M^{\text{cal}}(\omega_i, TE_l)]^2, \quad (8)$$

where the calculated values M^{cal} are provided by the representation in Eq. 6 or 7. A nonlinear optimization problem is solved for each pixel. Convergence, if obtained, may be to one of several multiple local minima. Consequently, multiple starting values at each pixel may be required in order to achieve the global optimum. In our experience, the stretched exponential model may not adequately represent the relaxation process (Chen et al., 1993). In principle, the multiexponential representation can represent the relaxation process if adequate numbers of terms are used. Suitable selection of the appropriate number of terms can be made through application of a statistical test at each pixel. As can be seen, considerable attention to the estimation problem can be required at each pixel. Such difficulties are avoided using the proposed technique presented next.

Relaxation Analysis by Continuous Distribution Model

Many porous media can be represented with a distribution of pores of various sizes. Under the fast exchange limit

(Brownstein and Tarr, 1977), relaxation in such porous media corresponds to a continuous distribution of relaxation rates. The magnetization intensity for a pixel, corresponding to a Hahn spin echo, would be given by

$$M^{\text{obs}}(\omega_i, TE) = \int_{T_{2\min}}^{T_{2\max}} P_i(T_2) \left\{ \exp\left(-\frac{TE}{T_2}\right) \right\} dT_2. \quad (9)$$

Once the unknown distribution $P_i(T_2)$ is estimated, the intrinsic magnetization intensity $M_0(\omega_i)$ at each pixel is calculated as

$$M_0(\omega_i) = \int_{T_{2\min}}^{T_{2\max}} P_i(T_2) dT_2. \quad (10)$$

Equation 9 is a Fredholm integral equation of the first kind, and the recovery of the distribution function from the observations is known to be an ill-posed problem. A method recently proposed to solve such problems (Liaw et al., 1995) can be applied here. In this method the unknown distribution $P_i(T_2)$ is represented using B-spline basis functions

$$P_i(T_2) = \sum_{j=1}^n c_{\omega_i, j} B_j^m(T_2). \quad (11)$$

A sufficiently large dimension of the spline space n is used so that the estimation of the distribution function is largely uninfluenced by the number and position of knots.

Regularization with a second derivative as the regularizing operator is used to solve the problem. This corresponds to determination of coefficients C that minimize

$$J(\lambda, C) = \|Y^{\text{obs}} - AC\|^2 + \lambda \|LC\|^2, \quad (12)$$

subject to nonnegativity constraints

$$c_j \geq 0, \quad j = 1, 2, \dots, n, \quad (13)$$

where

$$\|LC\| = \sum_{j=1}^n (c_{j-1} - 2c_j + c_{j+1})^2 \quad (14)$$

and A is the matrix representation obtained by substituting Eq. 11 into Eq. 9 for different TE values so that

$$a_{ij} = \int_{T_{2\min}}^{T_{2\max}} B_j^m(T_2) \exp\left(-\frac{TE_i}{T_2}\right) dT_2. \quad (15)$$

The largest value of λ that does not compromise the precision of the fit to the data is chosen as the optimal value for λ (Yang and Watson, 1991).

Results

The analysis is demonstrated using data obtained by imaging a Texas Cream limestone sample which is 2.54 cm in diameter and 7 cm in length. The sample is saturated with hex-

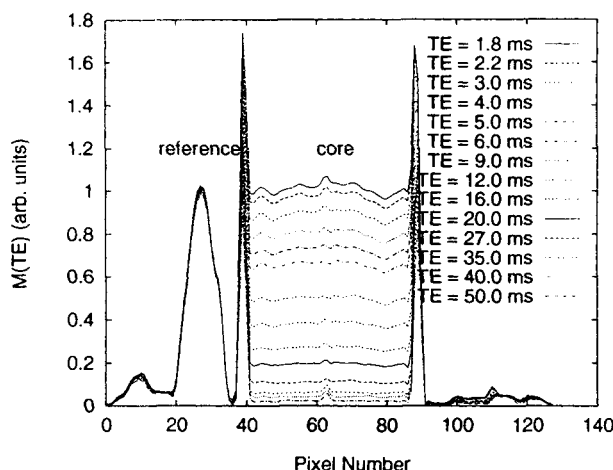


Figure 1. Profile images at several different TE values.

Spikes at the two ends of the core region are due to water trapped between end pieces of core holder and core. The reference signal does not relax as fast as the signal from the core due to the small relaxation rate of bulk water which is used as reference.

adecane and laterally sealed using epoxy (Stycast 2651) and mounted in a plexiglass coreholder which is used for displacement experiments. The sample is inserted in a GE 2-Tesla CSI-II imager/spectrometer with 31 cm magnet bore and equipped with 20 G/cm shielded gradient coil and a birdcage RF coil.

The images at different values of elapsed time TE (echo time) are shown in Figure 1. Each image is represented by 128 pixels in this example. The magnetization decay data for each pixel is used to model the relaxation process as described earlier in the section Background. The magnetization decay data for one of the pixels is illustrated in Figure 2. Also shown on that figure as a dashed curve are the corresponding decay values calculated using the continuous distribution model. As can be seen, the model provides a fairly precise representation of the measured data.

When using the continuous distribution model, a suitable value of the regularization parameter is to be chosen for each pixel. This is accomplished graphically by plotting the sum of

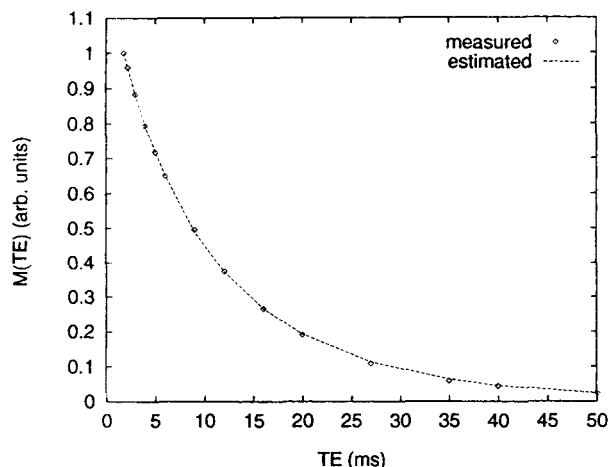


Figure 2. Magnetization decay at a pixel.

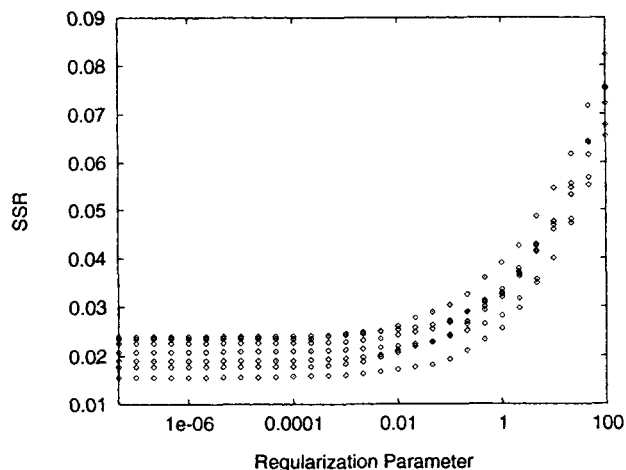


Figure 3. Sum of squared residuals for different values of regularization parameter λ and various pixels along the core.

squared residuals, $SSR = \|Y^{obs} - AC\|^2$, against λ . Since the data at all pixels and at all TE values are measured simultaneously, using the same device, the statistics of the errors in the data at each pixel are not expected to vary. Hence, the regularization parameter selected from the analysis of a randomly selected pixel can be used for all the pixels. This is verified with Figure 3 where SSR is plotted as a function of λ for several different pixels. For values of regularization parameter larger than 0.01, the SSR can be seen to increase and for values less than 0.01, the SSR does not decrease significantly. Hence, a value of 0.01 can be chosen for the regularization parameter for all pixels.

For a given value of regularization parameter λ , this minimization problem is a linear least-squares problem with linear inequality constraints. The solution, which is global, can be calculated in a direct manner as opposed to the iterative minimization procedures required with previous applications.

An important feature of the NMR profile images is that at each pixel the magnetization intensities are measured for the same set of TE values. This means that the matrix A in Eq. 12, the evaluation of which is the most computationally intensive part of the method, needs to be evaluated only once. This means that at each pixel, only a single linear least-squares problem with inequality constraints needs to be solved, using a different data vector at each pixel. This is a great advantage over the nonlinear methods used earlier.

Results obtained using both the multiexponential model and the continuous distribution model reported here are compared in Table 1 for two different samples. Average values of the porosity distribution are compared with the values obtained gravimetrically. It is seen that the values obtained

Table 1. Porosities Estimated by the Two Methods

Rock Sample	Continuous		Gravimetric
	Biexponential Model	Distribution Model	
Indiana Limestone	16.5 (2%)	16.4 (3%)	16.9
Texas Cream Limestone	24.2 (0%)	24.6 (2%)	24.2

with both models compare very favorably with the gravimetric values. However, the new technique has substantial advantages for convenience and reliability.

Conclusions

A new technique is presented for quantification of NMR imaging data that is particularly useful for observing fluids in porous media. The method is based on modeling the relaxation behavior with a continuous distribution of relaxation rates. The associated estimation problem is linear in the unknown parameters, so that the global optimum can be determined directly. The method offers considerable advantages in convenience, speed, and robustness compared to previous methods.

Notation

B_i^m = m th order B -spline function
 c_i = B -spline parameter
 C = vector of B -spline parameters
 L = regularization operator
 z = spatial dimension

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